

THE STRAIGHTFORWARD NUMERICAL TREATMENT OF THE TIME DEPENDENT ADVECTION IN AIR POLLUTION PROBLEMS AND ITS VERIFICATION

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Abstract—A very simple Lagrangian finite difference scheme has been developed to calculate the time dependent advection of air pollutants. It is mass conserving and avoids numerical pseudo-diffusion. No condition of numerical stability is required. The Eulerian grid used for the diffusion part of the pollutant transport equation remains unchanged. There are no restrictions on temporally and spatially variable emission rates, production and destruction processes, wind velocity, diffusion coefficients, roughness parameters or inversion heights. The only exception is that the wind field should not be too far from being homogeneous in the horizontal direction (test of D. W. Pepper and P. E. Long, 1978, *J. appl. Met.* 17, 228-233).

Steady state solutions are nearly identical with corresponding analytical solutions. The propagation of a pollutant cloud is simulated more realistically as compared with the advection treatment of E. Runca and F. Sardei (1975, *Atmospheric Environment* 9, 69-80) and M. Dunst (1980, *Z. Met.* 30, 47-59). The course of a diffusion experiment is modelled to demonstrate the efficiency of the proposed method.

Because of its simplicity, the method is especially suited for use in license processes, for control, and for calculating health risks in relation to industrial and power plant accidents with the goal of organizing efficient protection or evacuation.

1. INTRODUCTION

In most cases of practical relevance the time and space dimensions of plumes and puffs are of the order of or larger than the characteristic scales of the most effective turbulent elements, especially if we are not concerned with the immediate vicinity of the source. Then the governing equation for atmospheric diffusion problems of pollutants is

$$\frac{\partial c(\mathbf{x}, t)}{\partial t} = -(\mathbf{v}(\mathbf{x}, t) \nabla) c(\mathbf{x}, t) + \nabla(\mathbf{K}(\mathbf{x}, t) \nabla c(\mathbf{x}, t)) + \sum_i \frac{\partial cs_i}{\partial t}(\mathbf{x}, t) \quad (1)$$

where c denotes the concentration of the pollutant, $\mathbf{x} = \{x, y, z\}$ is the space vector, with x pointing to the east, y to the north and z vertically upward, t is time and $\mathbf{v} = \{u, v, w\}$ is the three dimensional wind vector. $\nabla = \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\}$ is the three dimensional delta-operator and ∂ means a local derivative. $\mathbf{K} = \{K_x, K_y, K_z\}$ is the coefficient of turbulent diffusion, and $\sum_i \frac{\partial cs_i}{\partial t}$ stands for changes by sources and sinks, chemical and photochemical processes, as well as radioactive decay. All dependent variables c , \mathbf{v} , \mathbf{K} , cs are functions of time and space.

With certain assumptions which simplify the physics significantly, analytical solutions of (1) have been obtained (see e.g. Smith, 1957; Rounds, 1955; Nieuwstadt, 1978; Sutton, 1953; Turner, 1967;

Pasquill, 1962). If the variables are space and time dependent, (1) must be solved by numerical techniques (see e.g. Egan and Mahoney, 1972; Runca and Sardei, 1975; Sheih, 1978; Shannon, 1979; Dunst, 1980; Lange, 1978; Sklarew *et al.*, 1971).

In this paper a very simple Lagrangian method of sufficient accuracy is presented which models the advection process adequately while being more economic than the methods mentioned above.

In section 2 some related work is reviewed, in section 3 the method is presented and section 4 contains the results obtained.

2. BRIEF DISCUSSION OF RELATED NUMERICAL SCHEMES

Essentially (1) consists of two parts:

$$\frac{\partial c}{\partial t} = \nabla(\mathbf{K} \nabla c) \quad (2)$$

can be solved without difficulty with sufficient accuracy by an Eulerian scheme.

$$\frac{\partial c}{\partial t} = -\mathbf{v}(\nabla c) \quad (3)$$

yields an artificial numerical diffusion if it is handled with an Eulerian scheme. Beside the pseudo-spectral method of Christensen and Prahm (1976), the Monte-Carlo technique (Bork and Maier-Reimer, 1978), and the methods using higher order difference schemes (Roberts and Weiss, 1966) two strategies have been

developed to overcome this pseudo-diffusion by a Lagrangian treatment of advection.

(A) First there are techniques known as particle-in-cell methods (Sklarew *et al.*, 1971), puff-in-cell methods (Sheih, 1978), conservation-of-moments models (Egan and Mahoney, 1972), and combinations of these (Shannon, 1979). Considering a non-divergent velocity field, (1) can be written in its flux conservation form

$$\frac{\partial c}{\partial t} + \nabla \cdot (c(\mathbf{V} + \mathbf{v}_D)) = \sum_i \frac{\partial c s_i}{\partial t}, \quad (4)$$

where $\mathbf{v}_D = -\frac{\mathbf{K}}{c} \nabla c$ denotes the diffusive velocity. The

crucial points are the representation of the concentration as puffs or particles and its rearrangement in neighbouring cells by assuming a Gaussian distribution or the conservation of the first and second moments. These techniques are well established in regional scale problems (Lange, 1978), although tests with rigid rotation wind fields (Shannon, 1979; Pepper and Long, 1978) reveal a considerable diffusion error.

(B) Secondly, there are techniques which calculate the contributions due to the diffusion terms (2) and advection terms (3) in different steps (Dunst, 1980; Runca and Sardei, 1975; hereafter called D and RS, respectively). Equation (2) is calculated by an Eulerian scheme and (3) is calculated by a Lagrangian technique. In both methods the given vertical wind profile $\mathbf{v}(z)$ is approximated by a step function \mathbf{v}_k , in such a way that the advection distance $\mathbf{v}\Delta t$ is equal to $n\Delta\mathbf{x}$, with $0 \leq n \leq q$, where q is an integer.

3. THE NUMERICAL SCHEME

For simplicity we restrict the problem to an inert pollutant, emitted by a single stack of effective height h , with emission rate Q . The wind blows horizontally and is a function of z only. Of course a vertical wind component and sedimentation of the pollutant could be handled likewise. The area downwind of the stack is flat but roughness may change. Then (1) takes the form

$$\begin{aligned} \frac{\partial c}{\partial t}(\mathbf{x}, t) = & -u(z)\frac{\partial c}{\partial x}(\mathbf{x}, t) - v(z)\frac{\partial c}{\partial y}(\mathbf{x}, t) \\ & + \nabla \cdot (\mathbf{K}(\mathbf{x}, t) \nabla c(\mathbf{x}, t)) + \frac{Q}{V}, \end{aligned} \quad (5)$$

where V is a volume which has to be specified. The continuous variables are decomposed on a not necessarily equally spaced spatial and temporal grid. For example:

$$c(x, y, z, t) \rightarrow c(i\Delta x, j\Delta y, k\Delta z, n\Delta t) \rightarrow c_{ijk}^n.$$

Using the method of fractional steps (Yanenko, 1971) (5) is split up into five steps: two diffusion steps (a horizontal and a vertical one), two advection steps, and a source term step, which are solved one after the other for each time step Δt .

3.1. The diffusion step

Depending on the problem ex- or implicit centered space difference schemes can be used. The handling of the explicit scheme (written down for the z -direction, a constant vertical grid spacing is assumed, indices i and j have been omitted, $K := K_z$)

$$\begin{aligned} c_k^{n+1} = & c_k^n + \frac{\Delta t}{(\Delta z)^2} [K_{k+\frac{1}{2}}(c_{k+1}^n - c_k^n) \\ & - K_{k-\frac{1}{2}}(c_k^n - c_{k-1}^n)], \end{aligned} \quad (6)$$

is very simple. For most purposes it will do since its accuracy is of the order $O(\Delta t) + O[(\Delta z)^2]$.

If a variable grid spacing is advantageous and/or if the diffusion time step

$$\Delta t_D \leq \min \left\{ \frac{(\Delta z)^2}{3K_z}, \frac{1}{6} \left(\frac{(\Delta x)^2}{K_x} + \frac{(\Delta y)^2}{K_y} \right) \right\}$$

which must be observed to guarantee numerical stability and to avoid oscillation is unprofitable short compared with Δt_A , the implicit Crank-Nicholson method

$$\begin{aligned} \frac{c_k^{n+1} - c_k^n}{\Delta t} = & \frac{2}{\Delta z_k(\Delta z_k + \Delta z_{k-1})} \{ bD(c_k^{n+1}) + (1-b)D(c_k^n) \} \end{aligned} \quad (7)$$

with

$$D(c_k) = K_{k+\frac{1}{2}}(c_{k+1} - c_k) - K_{k-\frac{1}{2}}(c_k - c_{k-1}) \frac{\Delta z_k}{\Delta z_{k-1}},$$

should be used which is stable for $\frac{1}{2} \leq b \leq 1$. Δt_A is the advection time step which may be chosen only from accuracy reasons, see 3.3. For further details see Richtmyer and Morton (1967).

Tests carried out with equal grid spacing and equal physical input show that the procedures (6) and (7) differed only by a few per cent in the cases studied in this paper. Therefore, (6) has been used throughout this paper.

3.2. Source term calculation

$$c_{i,j,k_s}^{n+1} = c_{i,j,k_s}^n + \frac{Q^{n+\frac{1}{2}}\Delta t}{\Delta x_s \Delta y_s \Delta z_s},$$

where $\Delta x_s \Delta y_s \Delta z_s$ is the volume of the box represented by the grid point (i, j, k_s) , which has the position of the effective stack height.

Two points have to be kept in mind:

(1) Numerical aspect: it is not crucial to distribute the effluent escaping the (small) stack top into the (large) grid box surrounding it, if the choice of the grid spacing is physically reasonable and the comments of 3.4. are observed and

(2) Physical aspect: the use of a Fickian-like diffusion parameterized by meteorological field variables, rather than by plume variables overestimates the diffusion in the vicinity of the stack until the scale of

the plume is of the order of magnitude of the most active turbulent elements of the surrounding air.

3.3. The advection step

For simplicity of representation we assume a westerly wind $u(z, t)$. We start with a particle being at a position $\mathbf{x}_0^n = \{x_0, y_0, z_0\}$ at time $n\Delta t$ representing the concentration of the surrounding box $\Delta x \Delta y \Delta z$. In reality, the particle will be transported to the position

$$\mathbf{x}^{n+N} = \left\{ x_0 + \int_0^{N\Delta t} u(z, t) dt, y_0, z_0 \right\}$$

during the time interval $N\Delta t$.

This physical process is actually simulated by the advection technique without any approximation of the vertical wind profile. This is essentially an improvement compared to the methods of D and RS concerning accuracy and saving of computer time.

Assume the initial position of the particle is (i_0, j_0, k_0) in the model at time step n . It stays at this position during M time steps until its distance

$$\sum_{m=1}^M u_{i_0, j_0, k_0}^{n+m-1} \Delta t,$$

from (i_0, j_0, k_0) is larger than $\Delta x/2$. However, its real position is kept in mind and the particle stays at its new position $(i_0 + 1, j_0, k_0)$ until it is advected by the real wind to a position closer to $(i_0 + 2, j_0, k_0)$.

Figure 1 may illustrate the procedure. The concentration of each box is represented by one particle. All particles of the level k_0 are shifted by the same distance. This procedure is likewise done with all particles of the other levels k appropriate to the velocities of those levels. If, by diffusion or advection, the pollutant has reached a neighbouring level, the particle representing that concentration is assumed to have the same position in its box as all the particles of that level. If the wind velocity at a level k varies with time, all particles of that level will move with the new speed. In this way it

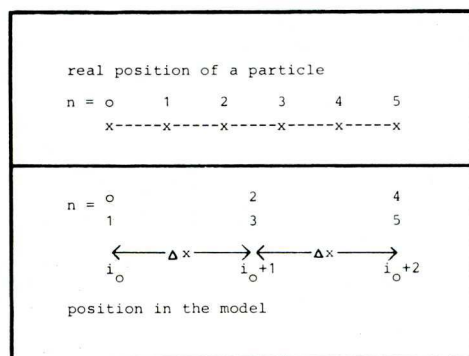


Fig. 1. Sketch of advection procedure. Crosses give the real position of a particle at time steps n labeled above. Numbers below mark the corresponding positions in the model at time steps n . Example: $u = 10 \text{ m s}^{-1}$, $\Delta x = 100 \text{ m}$ and $\Delta t = 4 \text{ s}$.

is achieved that in each box there will be only one particle. The method is unconditionally stable and the maximum advection error is $\Delta x/2$.

If $\Delta x \leq v_{\max} \Delta t$, the numerical routine is very simple. For this case a flow diagram of the advection process is shown by Fig. 2. Note that r ranges from 0 to Δx . If the wind field is homogeneous in the horizontal direction, the decision part has to be done only once for each vertical level k and each time step. This procedure is very economic regarding computer time and storage.

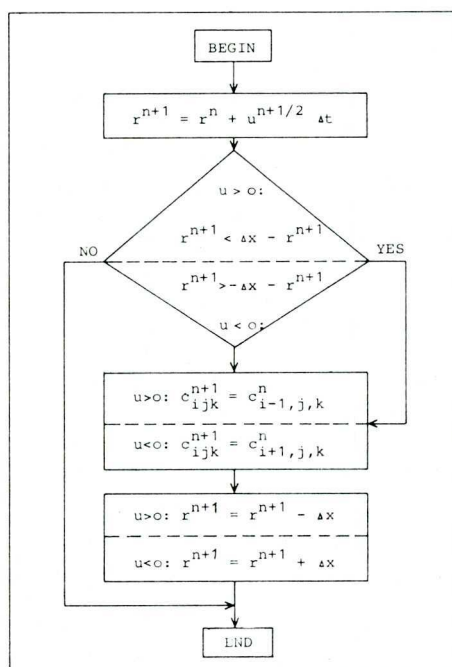


Fig. 2. Flux diagram of the advection procedure in x -direction for $u = u(z, t)$ at level k and time step $n+1$.

3.4. Handling of the method

In this section some hints at the application of the method are given. A wind field $\mathbf{v} = \{u(z, t), v(z, t), 0\}$ is assumed which is the most common one in practice. The initial data are the positions \mathbf{x}_{ijk}^0 of the particles and the concentration field c_{ijk}^0 . A value at grid point (i, j, k) represents a pollutant mass $c_{ijk} \Delta x_i \Delta y_j \Delta z_k$.

The course of the emission rate $Q(t)$ must be measured or derived from physical considerations. The velocity field $\mathbf{v}(z, t)$ must be measured or predicted by a boundary layer model (Dunst, 1980). The diffusion coefficients $\mathbf{K}(\mathbf{x}, t)$ are parameterized using meteorological variables which have been measured or predicted by a boundary layer model. The sequence of the fractional steps is of no importance if the space and time grids are reasonably chosen.

A "numerical Reynolds number" is defined giving a correct ratio of the grid element dimensions $\Delta x, \Delta y$ and

Δz . In order to get realistic results the ratios

$$\frac{u(\Delta z)^2}{3\Delta x K_z} \quad \text{and} \quad \frac{v(\Delta z)^2}{3\Delta y K_z}, \quad (8)$$

are allowed to exceed unity only at a few points of the concentration field.

Its meaning is that the characteristic time scale of the advection terms $\frac{\Delta x}{v}$ should be larger than that of the vertical diffusion term $\frac{(\Delta z)^2}{3K_z}$. Otherwise the diffusion of the pollutant is too quick through the vertical levels resulting in a not realistically high concentration at the surface. If the physical parameters v and K_z are given, this can be achieved by the choice of a small Δz (then the pollutant needs many time steps to reach the surface) or by choosing Δx large since in larger distances from the source the pollutant will actually have reached the surface. For this reason it is convenient to put an elevated source at least four vertical levels above the surface level.

The calculated concentration values might not be realistic in the vicinity of a point source if its effluent is distributed into the surrounding grid box. If no tests are carried out a distance of $4\Delta x$ is recommended where the numerical values should not be interpreted.

In general there exists a small oscillation for each combination $v_k, \Delta x_k, \Delta t$. If n_k and m_k are two integers such that

$$n_k v_k \Delta t = m_k \Delta x_k$$

holds for a level k , its wavelength is $L = m_k \Delta x_k$ and its frequency is $f = \frac{1}{n \Delta t}$. To eliminate this oscillation totally the mean over n time steps has to be taken.

However, the resulting truncation error in the concentration field is negligible. If the relevant parameter is the time integral of concentration (dose), it is of no importance at all. In the special case $n = m = 1$ no oscillation occurs.

Another oscillation originates from the stepwise shifting of all particles of a distinct level which may take place at another time step than the shifting at neighbouring levels. An example can be seen in Fig. 5(a) where the shifting has just taken place for the levels at $z = 0.32$ and 0.44 whereas the particles have stayed at their old grid points at $z = 0.36$ and 0.40 . This will be compensated at the next time step. The arising truncation error is too small to affect the usefulness of the advection procedure. Especially if the input data are not mathematical functions, but physical values which themselves are inaccurate, the truncation error is negligible.

4. RESULTS AND VERIFICATION OF THE METHOD

First the verification tests reported by RS are used for comparison purposes. Equation (5) then takes the form

$$\frac{\partial c(x, z, t)}{\partial t} = -u(z) \frac{\partial c(x, z, t)}{\partial x} + \frac{\partial}{\partial z} \left[K(z) \frac{\partial c(x, z, t)}{\partial z} \right] \quad (9)$$

in connection with (11).

The initial condition is

$$c(x, z, t) = 0 \quad \text{at } t = 0, \quad (10)$$

meaning that the region is not polluted at the onset of emission. The concentration at the source is assumed to be a δ -function

$$c(0, z, t) = -\frac{\delta(z-h)}{u(h)} \quad \text{at } t > 0. \quad (11)$$

The vertical boundary conditions are

$$K(z) \frac{\partial c}{\partial z} = 0 \quad \text{at } z = 0, 1, \quad (12)$$

describing a ground surface and an inversion layer which totally reflect the pollutant. Of course deposition velocities could easily be considered.

The numerical model needs horizontal boundary conditions, too.

$C = 0$ for inflow boundaries

$$\frac{\partial c}{\partial n} = 0 \quad \text{for outflow boundaries,} \quad (13)$$

where n is the direction normal to the boundary. If the normal component of the inertia terms is small compared to that of the diffusion terms the latter boundary condition is in general not physically realistic.

In the vertical direction the grid is arranged such that the lowest level $k = 1$ is $\frac{\Delta z}{2}$ away from the ground to easily fulfill condition (12).

4.1. Steady-state solution

Figure 3 shows a comparison between the analytical (Rounds, 1955) and the numerical steady-state solution. The height of the concentration maximum decreases with increasing distance from the source. The correspondence is very encouraging. Even for $x = 0.01$ which is only five grid points down stream of the source the accuracy is outstanding. This means that the distribution of a line (and point) source effluent into a grid box is not critical, even adjacent to the source with the parameters chosen.

Figure 4 presents the horizontal concentration profiles. The vertical concentration gradient decreases with increasing distance from the source. Again the more than satisfactory overall quality of the numerical solution can be seen.

Even in the case of the steady-state solution for continuous sources the proposed method is advantageous over the Lagrangian type advection treatment of D and RS and over the more conventional spectral and Eulerian techniques. In the latter methods the truncation error must be corrected by rearrangement

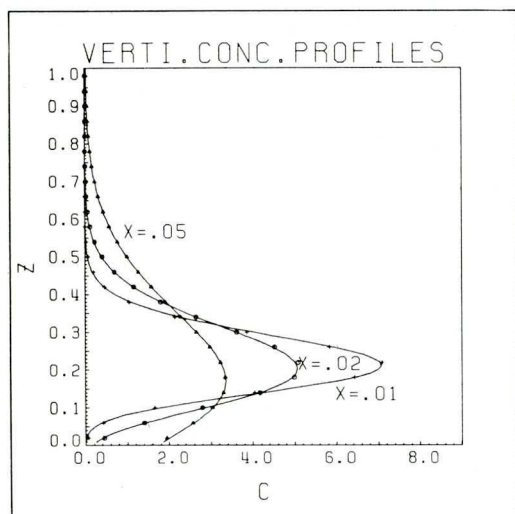


Fig. 3. Analytical and numerical steady-state vertical concentration profiles calculated for $u(z) = z^{0.2}$, $K(z) = z$, $h = 0.22$. Curves represent the analytical solution at distances $x = 0.01$, $x = 0.02$, $x = 0.05$ from the source. Discrete symbols denote the corresponding numerical solution. Parameters used are $\Delta x = 0.002$, $\Delta z = 0.04$, $\Delta t = 0.001$.

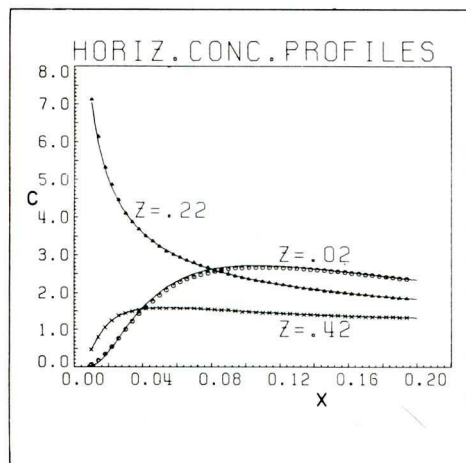


Fig. 4. Analytical and numerical steady-state horizontal concentration profiles. Curves represent the analytical solution at heights $z = 0.02$ (lowest level), $z = 0.22$ (source level), $z = 0.42$. Discrete symbols denote the corresponding numerical solution. All parameters used are the same as in Fig. 3.

of the pollutant in neighbouring cells or it is partially eliminated by higher order finite difference schemes. Both methods are time consuming.

The former methods need a larger computer time by a factor q to come up to an equivalent accuracy. q depends on the vertical wind gradient (see 2(B)). Comparing Fig. 3 of this paper with Fig. 4 of RS it can be seen that for the example chosen the accuracy of the method presented here is superior although RS used a denser horizontal grid by a factor $q = 5$ and, moreover, a variable vertical grid in order to provide a better resolution near the source.

4.2. The front of a travelling cloud

In Figs 5(a) and 5(b), time sequences of pollutant distribution are shown at times $t = 0.04$ and $t = 0.12$, respectively. Because of the vertical wind shear, the cloud front speed is slow at the ground leading to an accumulation of pollutants which is larger and nearer to the source than in the case of a constant velocity $u = \bar{u}$. The increasing diffusion coefficient distributes the effluent more quickly to greater heights than a constant $K_z = \bar{K}_z$ would do. Adjacent to the source, steady-state concentration has already been established at $t = 0.04$. The lines of constant concentration were only calculated to $\frac{\Delta z}{2}$ because of the height of the lowest grid level. The deformation of the iso-concentration line $c = 0.125$ at the top at $t = 0.12$ comes from the vertical boundary condition.

The vertical slope of the pollutant front resembles very well that of the vertical wind profile as opposed to the methods of D and RS, see Fig. 5(c) and 5(d). Comparison of Fig. 5(a) and 5(d) clearly demonstrates the effect of the step function used by RS: Between $z = 0.25$ and 0.40 the cloud moves too slowly and has not the correct slope. The use of a step function v_k gives rise to an advection error

$$x_e = (v(z) - v_k)n\Delta t,$$

where n is the number of time steps. Thus the prediction error of the arrival time of a pollutant cloud increases with travel time. If the advection procedure proposed in this paper is used, x_e is always smaller than $\frac{\Delta x}{2}$

4.3. Modelling of a tracer experiment

To demonstrate the efficiency of the proposed method, a diffusion experiment is modelled. The concentration in the neighbourhood of a plant or another emittent can be calculated on line in the same way giving valuable information as to where to make measurements or to evacuate people after accidents.

At the site of the West German Nuclear Research Center Karlsruhe, more than 70 tracer experiments have been done to evaluate the parameters σ_y and σ_z as needed in Gaussian formulas (Thomas *et al.*, 1976). The roughness length of the experimental area is $z_0 = 1.1$ m, due to buildings and a wood. The meteorological data are 10-min means taken at several heights from the Center tower of height 200 m (Table 1) from which the tracers were emitted with constant rate at a height of 100 m. The measured 30-min ground-level doses are compared with calculated accumulated concentrations at the lowest grid level $\frac{\Delta z}{2}$.

To make the experimental data acceptable for the model, the following simple adjustments were made. Temperature T , wind speed w and wind direction were interpolated linearly with time and to the heights

corresponding to the levels of the model. The wind speed was extrapolated to the ground by the logarithmic wind profile. The wind direction in the first 40 m was equal to that at $z = 40$ m. Resolution was $\Delta x = \Delta y = 75$ m, $\Delta z = 20$ m, $\Delta t = 10$ s.

In the last column of Table 1 values of K_z are

specified. K_z was parameterized by (Wu, 1965)

$$K_z = l^2 \left(\left(\frac{dw}{dz} \right)^2 - \frac{g}{T} \frac{d\Theta}{dz} \right)^{\frac{1}{2}}, \quad (14)$$

where l is the characteristic scale of the most effective eddies, g is the constant of gravity and Θ is potential

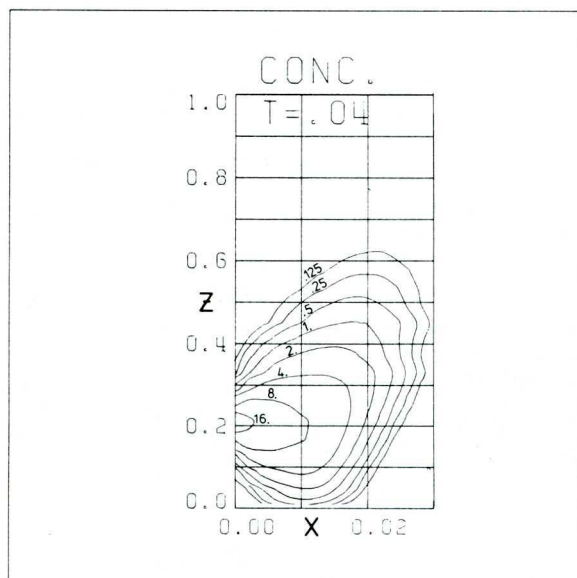


Fig. 5(a). Concentration in $x-z$ plane calculated for $u(z) = z^{0.5}$, $K(z) = z$, $h = 0.22$ for $t = 0.04$.

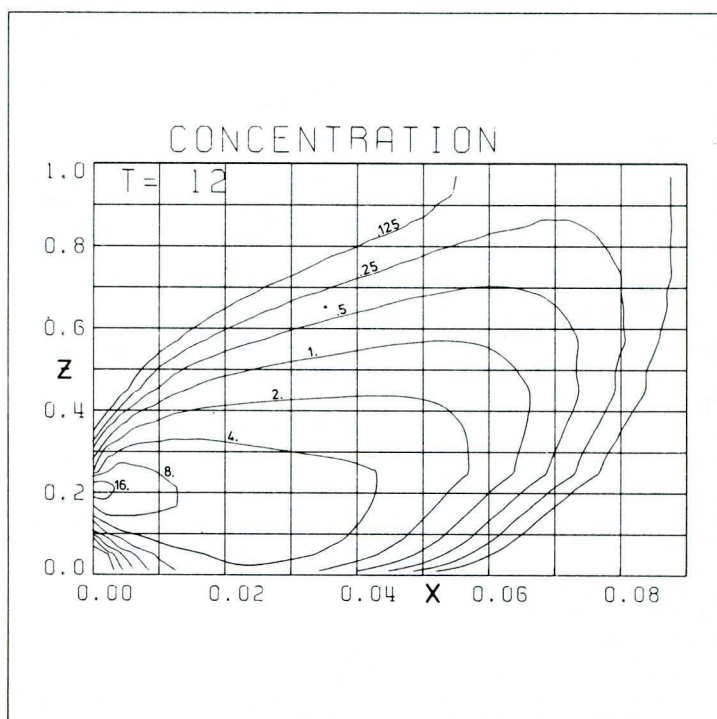
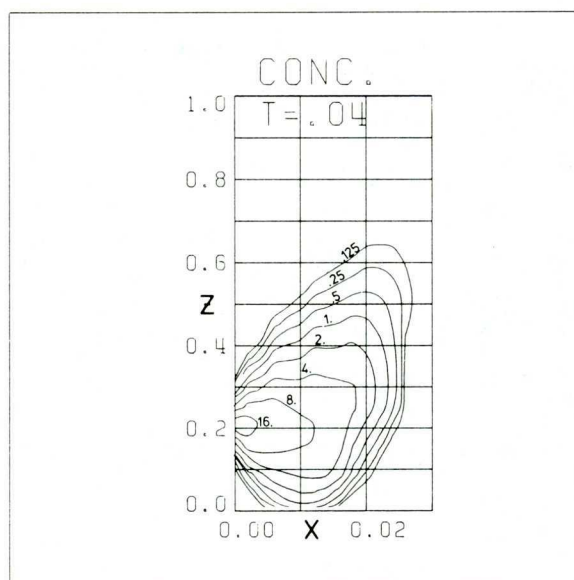
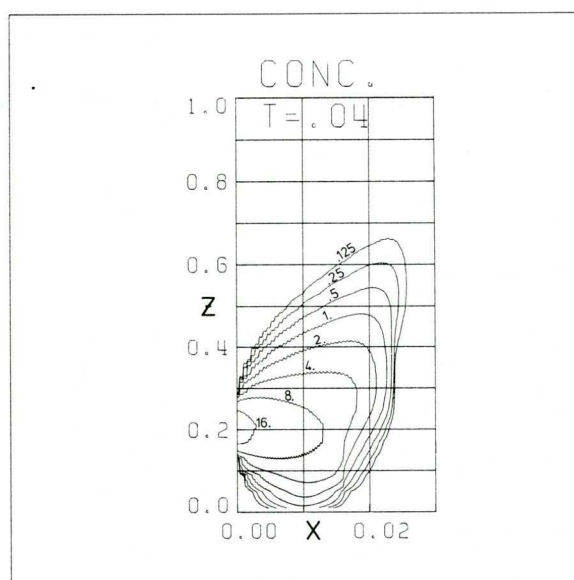


Fig. 5(b). As Fig. 5(a), but for $t = 0.12$.

Fig. 5(c). As Fig. 5(a), but with method of Dunst (1980), $q = 5$.Fig. 5(d). As Fig. 5(a), but with method of Runca and Sardei (1975), $q = 5$.

temperature; l was taken after Ohmsted and Appelby (1964):

$$l = l_0 [1 - \exp(-k_0(z+d)/l_0)]$$

where l_0 depends, among other factors, on friction velocity (Blackadar, 1962). Here $l_0 = 30$ m (see Wu, 1965), k_0 is von Kármán's constant, $d = 10$ m is a displacement height depending on the size of the ground obstacles.

The vertical slope of K_z is not as smooth as might be expected from theoretical considerations. However,

the values of K_z are reasonable, small adjacent to the ground, and have a maximum between 70 and 160 m. Moreover, K_z changes considerably during 10-min intervals. Nevertheless, according to Fig. 6 the calculated concentration field is quite realistic.

Values of the horizontal diffusion coefficient K_H are very uncertain. First calculations with

$$K_H = f(\sigma_v) K_z, \quad (15)$$

are encouraging. σ_v is the standard deviation of the 10-min mean of the horizontal wind direction.

But in this paper it was only intended to demonstrate the usefulness of the advection scheme. Furthermore, with a characteristic value of $K_z = 15 \text{ m}^2 \text{ s}^{-1}$ and $w = 6 \text{ m s}^{-1}$, see Table 1, the horizontal spread of the tracer is of the order

$$\sigma_y = (2K_z x/w)^{1/2} = 100 \text{ m},$$

if $K_z = K_H$ and $x = 2000 \text{ m}$. In view of this consideration σ_y exceeds the horizontal grid spacing $\Delta x = \Delta y = 75 \text{ m}$ only slightly. Therefore K_H has been neglected. No effort has been made to optimize the result by tuning the free parameters. This (and the use of other formulas for K_z) will be the next task.

5. CONCLUSIONS

A very simple Lagrangian advection technique, economic in computer time and storage, has been presented. In combination with an Eulerian diffusion scheme, it is particularly applicable for license processes, safety analyses, surveillance, implementation of control measuring nets and (in an on line version) for giving information to minimize health risks after industrial or (nuclear) power plant accidents. Furthermore, the effect of industrial areas on the neighbouring region can be calculated. The only restriction is that the wind field should not be too far from being homogeneous in the horizontal direction. The accuracy of the numerical calculations compared with the analytical solutions is very reasonable if the "numerical Reynolds numbers" are taken into account. Time dependent emission rates, wind fields and diffusivity, as well as low wind speeds can easily be simulated, making this method superior to "Gauss-models". In addition, this simple numerical method presented above shows the same efficiency and comparable accuracy to more complicated methods mentioned in the introduction. To make it a valuable tool in air pollution problems, the diffusion parameterisation has to be calibrated by the results of diffusion experiments.

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